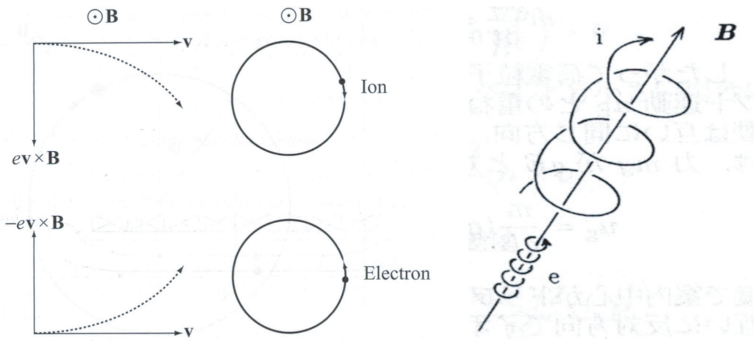
0 Magnetically Confined Plasma

0.1 Particle Motion in Prescribed Fields

The equation of motion for species s (= e, i) is written as

$$m_s \frac{d\vec{v}_s}{dt} = q_s \left(\vec{E} + \vec{v}_s \times \vec{B} \right).$$

The motion in a constant magnetic field $\vec{B}_0 = B_0 \hat{z}$ consists of the circular cyclotron motion (also called the gyromotion) about a magnetic field line and the motion of the center of the gyromotion (guiding center) along a magnetic field line. The angular frequency of the gyromotion is called the cyclotron frequency $\Omega_s \equiv q_s B_0 / m_s$ and the radius of the gyromotion is called the Larmor radius $r_{\mathrm{L}s} = v_{\perp s} / |\Omega_s|$.



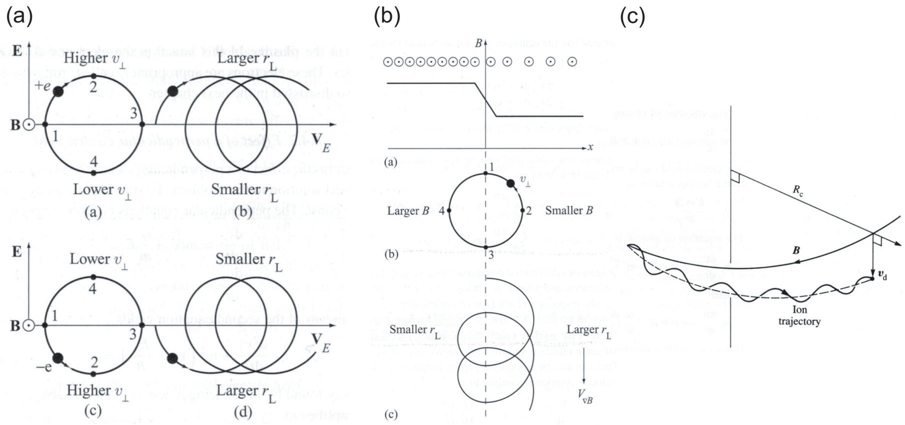
Charged particle motion in a constant magnetic field.

In the presence of inhomogeneities, such as the electric field (gradient in electrostatic potential), magnetic field gradient, and magnetic field curvature, the guiding center undergoes slow spatial drifts in addition to the gyromotion.

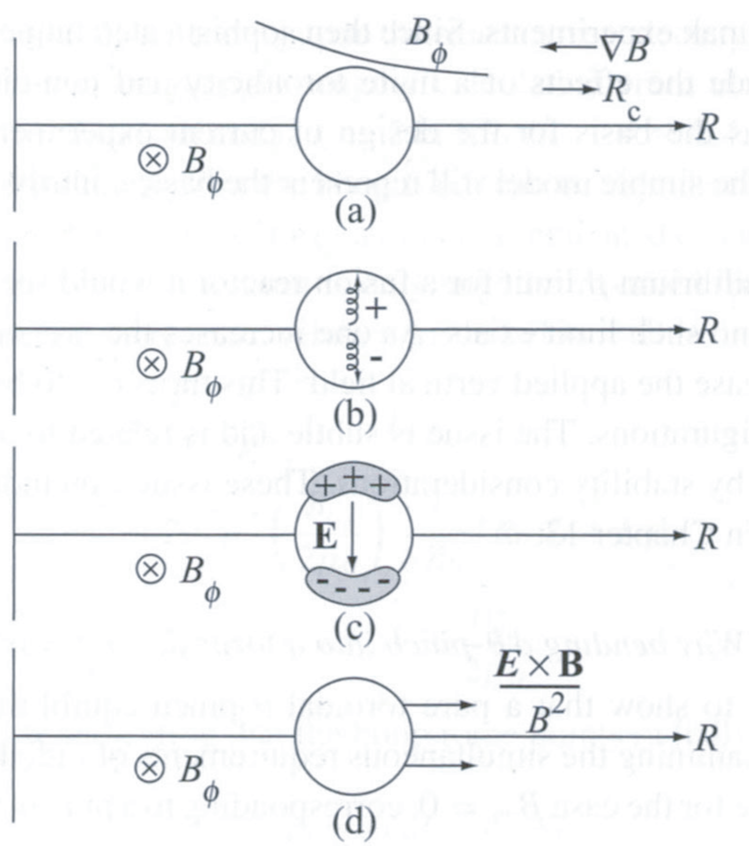
Since charged particles move primarily along a magnetic field line, it seems reasonable to confine charged particles in a configuration with circular (toroidal) magnetic field lines, B_ϕ. However, charged particles cannot be confined in a torus with only B_ϕ, since such a configuration inevitably has both magnetic field gradient and curvature. The ∇B drift and the curvature drift cause charged particles with different signs to drift in opposite directions, and cause charge separation. The charge separation induces the electric field in the vertical direction, and causes both ions and electrons to drift outward by the $\vec{E} \times \vec{B}$ drift. In order to prevent charge separation, a magnetic field in the poloidal (circumferential in the cross section of a torus) direction must be added.

0.2 Self-Consistent Fluid Model

The motion of charged particles is governed by the equation of motion, and are affected by both electric and magnetic fields. Conversely, the charged particle



Charged particle drifts: (a) $\vec{E} \times \vec{B}$ drift, (b) ∇B drift, (c) curvature drift.



Charge separation and outward charged particle loss in a torus with only B_ϕ.

distribution and motion affect the electric and magnetic fields. Therefore, the charged particle motion and the electric and magnetic field evolution must be determined self-consistently. There are three conservation equations for each speies (electrons, ions): mass conservation, momentum conservation, and energy conservation, and the electric and magnetic fields are governed by the four Maxwell equations. The charge density and the current density needed in Maxwell equations are given by the number densities and flow velocities of charged particles. These ten equations must be solved self-consistently.

0.3 MHD Equilibrium

The following set of equations, called the MHD (magnetohydrodynamic) equations, are derived by combining the electron and ion conservation equations.

$$\begin{aligned} \frac{d\rho}{dt} + \rho\nabla\cdot\vec{V} &= 0 \\ \rho\frac{d\vec{V}}{dt} &= \vec{j}\times\vec{B} - \nabla p \\ \vec{E} + \vec{V}\times\vec{B} &= \eta_{\|}\vec{j} \\ \frac{d}{dt}\left(\frac{p}{\rho^{\gamma}}\right) &= 0 \end{aligned}$$

These are called, respectively, the mass conservation equation, the momentum conservation equation, Ohm 's law, and the energy conservation equation.

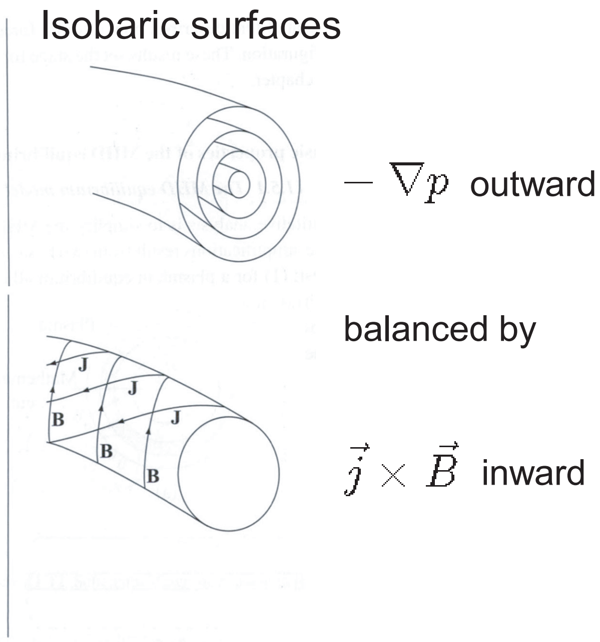
The static MHD equilibrium is obtained by setting the time derivative and the flow velocity to zero. The equations to be solved are reduced to

$$\begin{aligned} \vec{j}\times\vec{B} &= \nabla p \\ \nabla\times\vec{B} &= \mu_0\vec{j} \\ \nabla\cdot\vec{B} &= 0 \end{aligned}$$

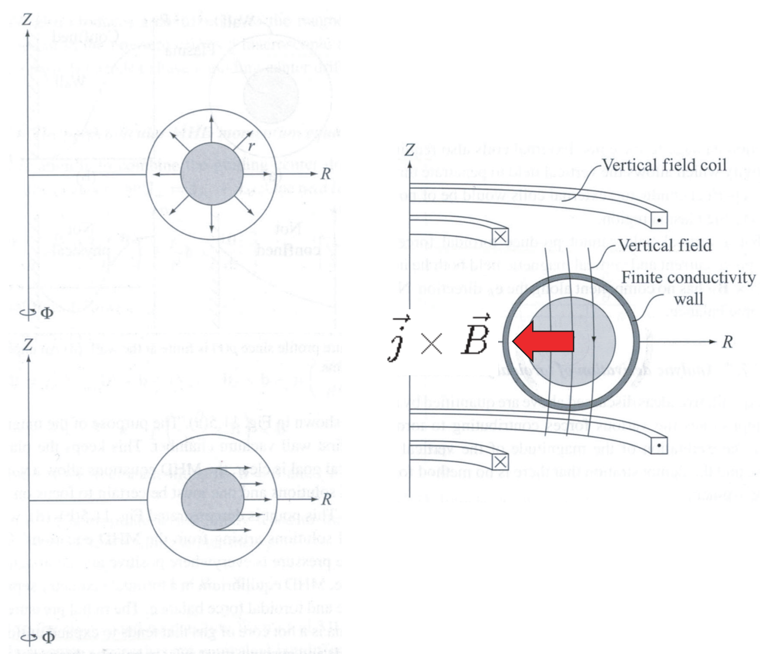
From the first equation, it follows that both $\vec{j}$ and $\vec{B}$ lie on isobaric (constant pressure) surfaces, which are nested toroidal surfaces. The pressure gradient force $-\nabla p$ is outward in the minor radial direction. This force must be balanced by the inward directed $\vec{j}\times\vec{B}$ force.

0.4 Tokamak

The tokamak is the most successful magnetic confinement configuration with very good confinement and adequate stability. It has already achieved high performance plasmas (high enough density, temperature, and confinement time) needed to demonstrate scientific breakeven, and a next-step device to study fusion burning plasmas with fusion output power as much as ten times the heating input power is being constructed by international collaboration. In the tokamak, very strong toroidal magnetic field is created by currents in the toroidal field coils, whereas the poloidal magnetic field is created by the current

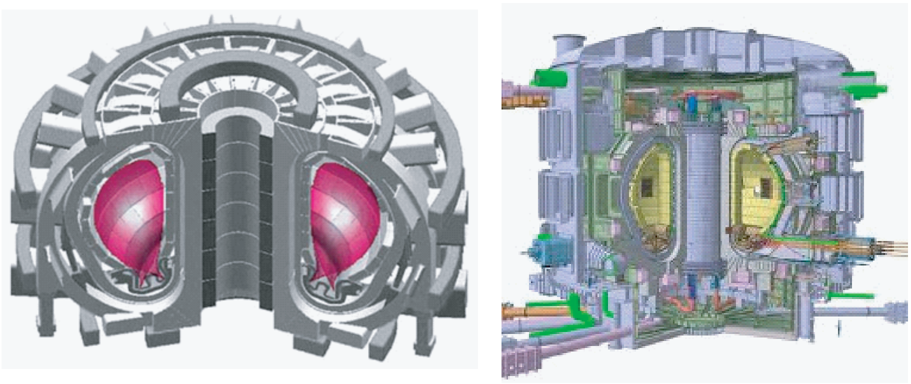


Static MHD equilibrium.



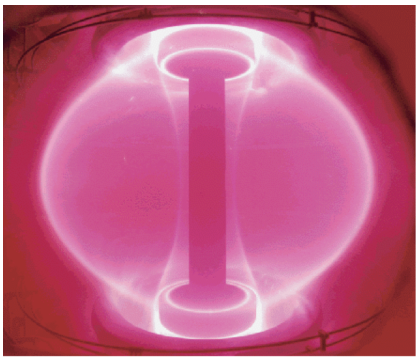
Equilibrium force balance. The vertical magnetic field must be added to provide the inward directed $\vec{j} \times \vec{B}$ force in the major radial direction to balance the outward directed hoop force.

driven inside the plasma. Additional poloidal field coils provide the vertical magnetic field needed for toroidal force balance and control the plasma cross sectional shape, which is important for improving stability and for exhausting heat and particles.



Typical coil configuration of the tokamak, and the International Thermonuclear Experimental Reactor (ITER) being constructed to study fusion burning plasmas.

The conventional tokamak has the disadvantage that it requires large external power to drive the plasma current needed for equilibrium. In the advanced tokamak operation mode, most of the plasma current is self-driven by the plasma. Since operation at high β (ratio of the plasma pressure to the magnetic pressure) is required, complicated systems are needed for profile control and active stabilization. The spherical tokamak, with very small ratio of major radius to minor radius (typically 1.5) has proven superior stability at high β (as high as 40%, which is three times higher than the highest β achieved in conventional tokamaks). It remains to be seen whether the spherical tokamak can be extended to the harsh environment of burning plasmas or not.



Photograph of a spherical tokamak plasma (START in UK).