Instrumentation and Information Processing　Homework No.1 for the first half part

You can obtain this instruction from http://fusion.k.u-tokyo.ac.jp/~ejiri/ejiri/report2019_1E.pdf

Dead line: May 10th, Send the report (in PDF) to ejiri@k.u-tokyo.ac.jp. Do not attach the program.

Perform Fourier transform for the following three data, calculate the power spectra, and plot them (seethe following plots. Confirm the Parsval's law.

(1) Make the following data, and plot them (see Fig.1)

Number of samples (data points) : 16, Time window : $1s$, Sampling time : $\frac{1}{16} = 0.0625$ s

Time $t = 0, \frac{1}{16}, \frac{2}{16}, \ldots, \frac{15}{16} = 0, 0.0625, 0..125, \ldots, 0.9375$

Signal　$y_1(t) = 1$　([V])

Singnal $y_2(t) = \sin(2\pi \times 2 \times t)$　([V])

Signal $y_3(t) = \cos\left(2\pi \times \frac{3}{2} \times t\right)$　([V])

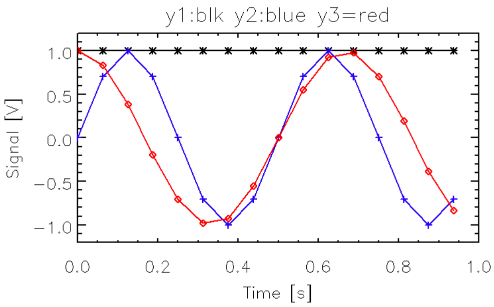

Fig. 1 Time evolutions of the signals y_1, y_2, y_3.

(2) Calculate the power spectral densities of y_1, y_2, y_3, and plot them with a logarithm scale vertical axis (see Fig. 2). The following shows some hints.

Since the time window is 1s, the frequency step (i.e., fundamental frequency) is Δf=1 Hz. The Nyquist frequency is 16/2=8 Hz, and the frequencies are 0, 1, 2, ..., 8 Hz as a result. The power is proportional to the square of the amplitude of the Fourier components. There are cosine and sine components for the frequencies: 1, 2, ..., 7 Hz, while 0 Hz is a DC component. Consider only the cosine component for 8Hz because it is the Nyquist frequency.

Power spectral density is the power per unit frequency.

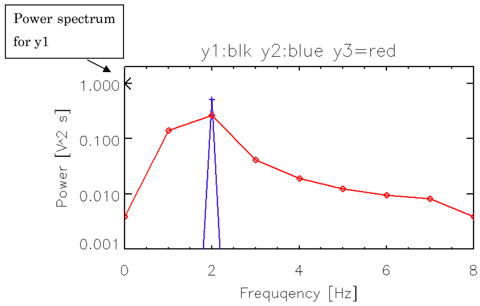

Fig.2 Power spectral density fo y_1, y_2, y_3.

(3) Confirm the Parsval's law, by calculating the mean square of the signal

$\langle y_1^2 \rangle = 1$　V^2、$\langle y_2^2 \rangle = 0.5$　V^2、$\langle y_3^2 \rangle = 0.5$ V^2,

and the total power, which is the integral of the power spectral density in frequency phase

$\int P(f)df = \langle y^2 \rangle$.

Confirm for the all data;　y_1, y_2, y_3.

Reference

Library (tools)

The libraries for Fourier transform may have different definitions and arguments, and you must be careful when you calculate the power.

Many libraries perform exp(iωt), exp(-iωt) transform instead of cosine and sine transform. In the former case, the typical order of output components are as follows 0, $\Delta\omega$, $\Delta\omega$,... ,(N/2) $\Delta\omega$, −(N/2−1) $\Delta\omega$,.....,−$\Delta\omega$, where $\Delta\omega$ is the fundamental frequency (i.e., frequency step).

FFT (Fast Fourier Transform)

While the calculation amount of the standard Fourier transform is proportional to N^2に, the Fast Fourier transform requires much smaller calculation amount of N logN. Note that the number of data points should be the product of small prime number. Read the instructions for each library.

Window function

Since the signal y_3 has a jump at t=0, t=1 s, high frequency components may be enhanced by the Gibb's phenomenon. In order to avoid such problems, apply a window function (see Figs. 3 and 4). Due to the window function, the amplitude near both ends become small, and Parsval's lawmay be distorted.

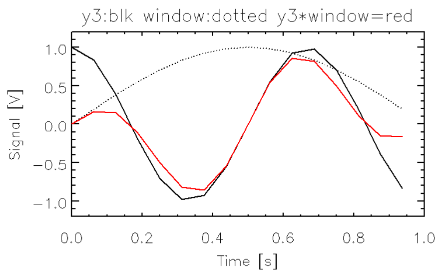

Fig. 3 Time evolutions of y_3 (Sloid), window function (dotted), and y3 with the window function (red).

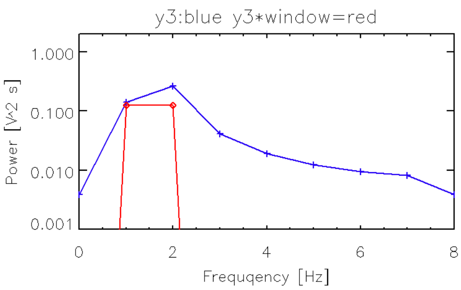

Fig.4 Power spectral density of y_3 (Blue) and that with a window function (red).